GENERALIZED SHAPE AND TOPOLOGY OPTIMIZATION: RECENT DEVELOPMENTS AND APPLICATION PERSPECTIVES TO AUTOMOTIVE STRUCTURES

P. Duysinx(1), L. Van Miegroet(1), A. Remouchamps(2) and C. Fleury(3)

(1) LTAS Automotive Engineering / (3) LTAS Multidisciplinary Optimization, University of Liège, Institute of Mecanics, B52, Chemin des Chevreuils, 1, BE-4000 Liège

(2) Samtech SA, Science Park, Rue des Chasseurs Ardennais, 8, BE-4031 Angleur, Belgium.

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1 Introduction

1.1 Optimization in engineering

Automotive engineering faces challenges in a very competitive world. To follow and to anticipate customers' expectations as fast as possible, car manufacturers have to release better models with higher performances, but they have also to shorten time-to-market every time. In this very competitive environment, optimization techniques provide engineers with automatic and rationale design tools.

At first optimization tools can nicely carry out reanalysis task, relieving engineers from the load of iterative process management. Then "trial and error" procedure is replaced by an efficient and rationale methodology based on mathematical programming algorithms, whose convergence properties guarantee steadily improvement of designs. Finally optimization provides a rationale tool to handle large and complex design problems with sometimes many thousand design variables with intricate direct and indirect influences. Mathematical strategy allows engineers to tackle complicated problems with strongly conflicting constraints, in which finding a feasible design is sometimes quite impossible for human understanding.

1.2 Optimization formulation of design problem

In the light of optimization theory, solving a design problem can always be cast as a constrained minimization / maximization problem. One is looking for minimizing one or several design criteria, called *objective functions*, while satisfying design *restrictions* treated as inequality or equality constraints. *Design variables* are the subset of parameters that the engineer chooses (or is allowed) to modify. Thus foundation of automatic design relies on the fact the design problem is stated as the following optimization problem:

$$\begin{array}{lll} \min\limits_{x_i} & f_l(x) & 1 \le l \le o \\ s.t. & g_j(x) \le \overline{g}_j & 1 \le j \le m_i \\ & h_k(x) = \overline{h}_k & 1 \le k \le m_e \\ & \underline{x}_i \le x_i \le \overline{x}_i & 1 \le i \le n \end{array} \tag{1}$$

Optimization formalism gives the opportunity to tailor a general and an open approach to many engineering problems. This approach has been successfully extended from structural optimization to multidisciplinary and now to multiphysic applications.

1.3 Solution of optimization problems using sequential convex programming

Solution of optimization problems (1) is very difficult because of the implicit and non linear character of the response functions (objective and restrictions) considered and because each function evaluation requires the execution of one heavy simulation of the problem (using finite element method FEM for instance). However for thirty years an efficient approach, called *sequential convex programming* [FLE03a] has been developed to handle structural and multidisciplinary optimization problems. It relies on two concepts:

Structural approximations: replace the implicit problem by an explicit optimisation sub-problem using convex, separable, conservative approximations; e.g. CONLIN (CONvex LINearization), MMA (Method of Moving Asymptotes)

- Solution of the convex sub-problems: efficient solution using dual methods algorithms or SQP (Sequential Quadratic Programming) method or even meta-heuristic algorithms like genetic algorithms.

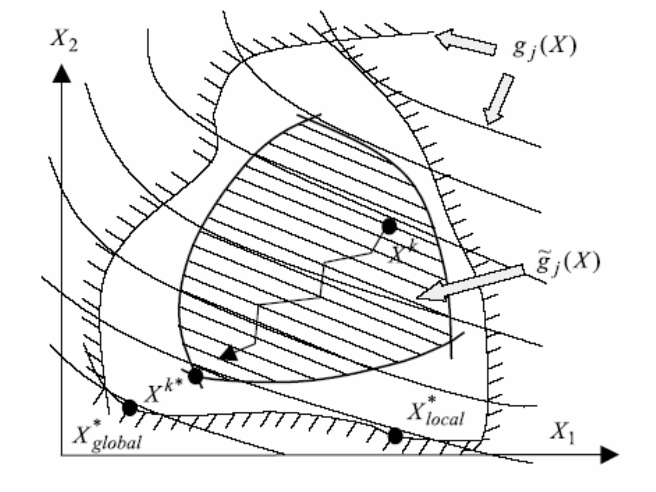

Figure 1: Sequential convex programming approach

Sequential convex programming has got several advantages. The first one is that optimized designs are reached generally within rather small number (e.g. 20) of reanalyses (FE simulations) so that design requires a restricted computation time. Second the method is efficient, robust, general and flexible: With little work the same procedure can be customized to solve many different problems including static, dynamic problems, composite materials, mechanism synthesis or multidisciplinary and now multiphysic problems. Finally the approach is able to solve large scale problems in terms of number of design variables (up to 500.000 design variable in topology optimization) but also a thousand of constraints (about 100.000 constraints among which 5.000 active constraints like in composite optimization). The CONLIN solver developed by Prof. Fleury [FLE03a] is one of the best available tools for research and industrial applications. It is used successfully for each of the following applications.

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2 Topology optimization

2.1 Formulation of topology optimization

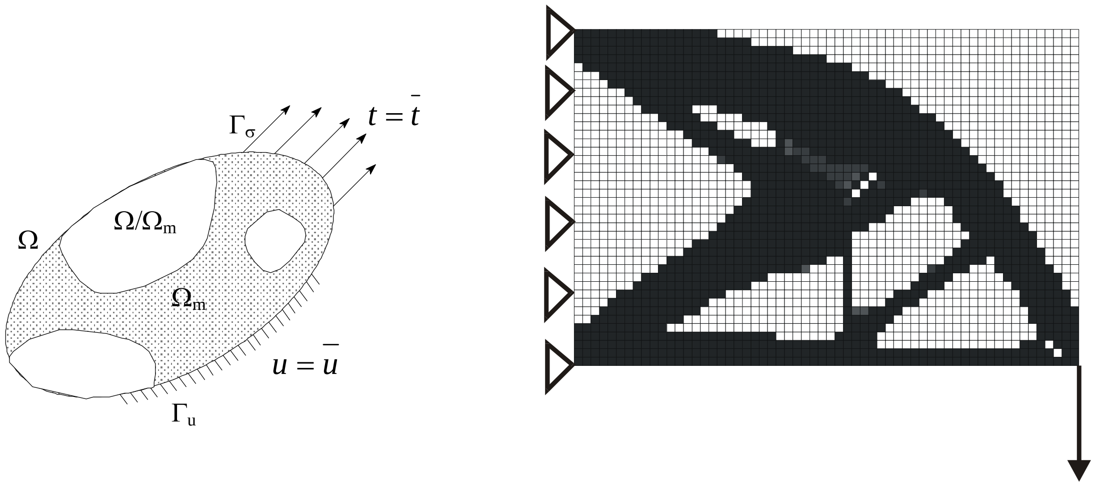
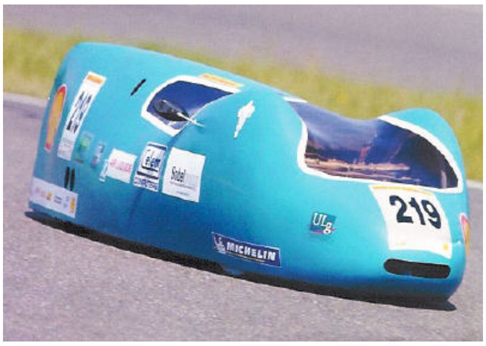


Figure 2: Topology optimization as an optimal material distribution

Preliminary design of automotive structures can be carried out using topology optimization approach. In order to determine the nature and the relative position of the structural members and holes, one has to get rid of classical CAD representation. As suggested initially by Bendsøe and Kikuchi [BEN88] the image of the optimum structure can be determined as an optimal material distribution. Presence or absence of material is indicated by density variables ranging from 0 (void) to 1 (solid). Even if the problem can also be solved using discrete variables as in Beckers [BEC99], one usually considers continuous density variables in the interval [0,1] while intermediate densities are penalized using for instance a power law penalization of material properties in order to take benefit of efficient mathematical programming algorithms [BEN03]. When the problem is discretized using finite elements, optimization problem attaches one or several design variables to each element introducing a large number of design variables. Because of this intrinsic large scale formulation, classical optimization problem is based on minimizing global (generally energy based) stiffness criteria subject to volume constraints. For a single load case, the optimum material distribution also gives a nearly fully stressed design. Nonetheless, Duysinx and Bendsøe [DUY98] showed later the local stress constrained problem could also be treated in a similar way but at the price of a much higher numerical cost. One major drawback of the material distribution formulation is the difficulty to introduce geometrical constraints on the shape. For instance it is rather difficult to control the minimum and maximum size of the members. Successful applications are only reported in controlling the complexity of the topology through bound upon the perimeter of the design or using casting constraints in order to avoid closed cell holes. Finally one has to observe the fast application of topology optimization into car industry design process, collecting now numerous success stories all over the world.

2.2 Optimum reinforcement of shells



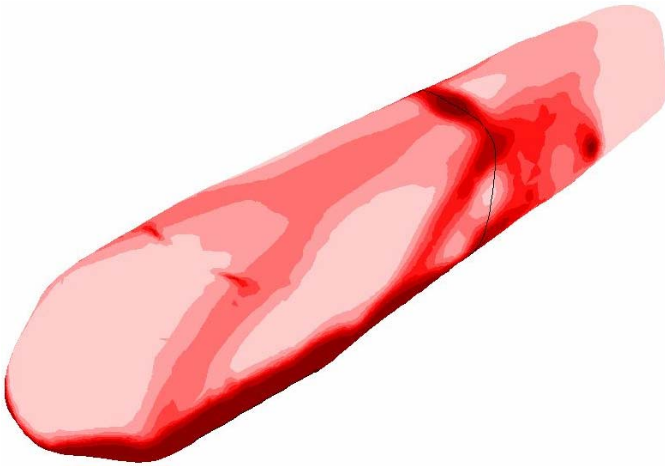


Figure 3: Topology optimization of a composite shell for an ultra light race car

The first application of topology optimization is dealing with the preliminary design of a composite shell for an ultra light Eco Marathon prototype vehicle. The aim of the Eco Marathon competition is design a car with the smallest fuel consumption[1] while making 7 laps (about 25 km) of the Nogaro race track with a minimum average speed of 30 km/h. University of Liège has raced for 3 years with a fuel cell powered prototype, finishing three times in a row in the top fifteen best teams over 250 competitors (see Fig 3 left). For this application, the structural weight is critical and must be minimized in order to maximize the fuel efficiency. The design of the new car body for 2007 season was studied using topology optimization. Material is a high modulus carbon epoxy composite. In the preliminary design phase a quasi isotropic stacking sequence is chosen: [0°/90°/+45°/-45°]. The considered load cases are the following: 1/ a shock against a curb with a dynamic factor 3 times the weight; 2/ a normalized roll-over load case with a 70 kg load on the top of the driver seat. After some investigations, we also considered bending and torsion stiffness using constraints upon the free-free eigenvalues of the body. Several eigenvalues were also taken into account in order to avoid undesirable vibrations of panels. As illustrated in Fig. 3 right, topology optimization provided a great support for design of the body shell. One can distinguish easily the front and the two side windows. But the most spectacular is that the weight of the body shell could be reduced to 12 kg leading to substantial fuel consumption: In a further design step a CAD model of the vehicle was built getting its inspiration from topology optimal material distribution. Simulation can predict fuel consumption over 2000 km per equivalent litre of gasoline.

[1] http://www.shell.com/home/Framework?siteId=eco-marathon-en

2.3 Stiffness and strength optimum topology

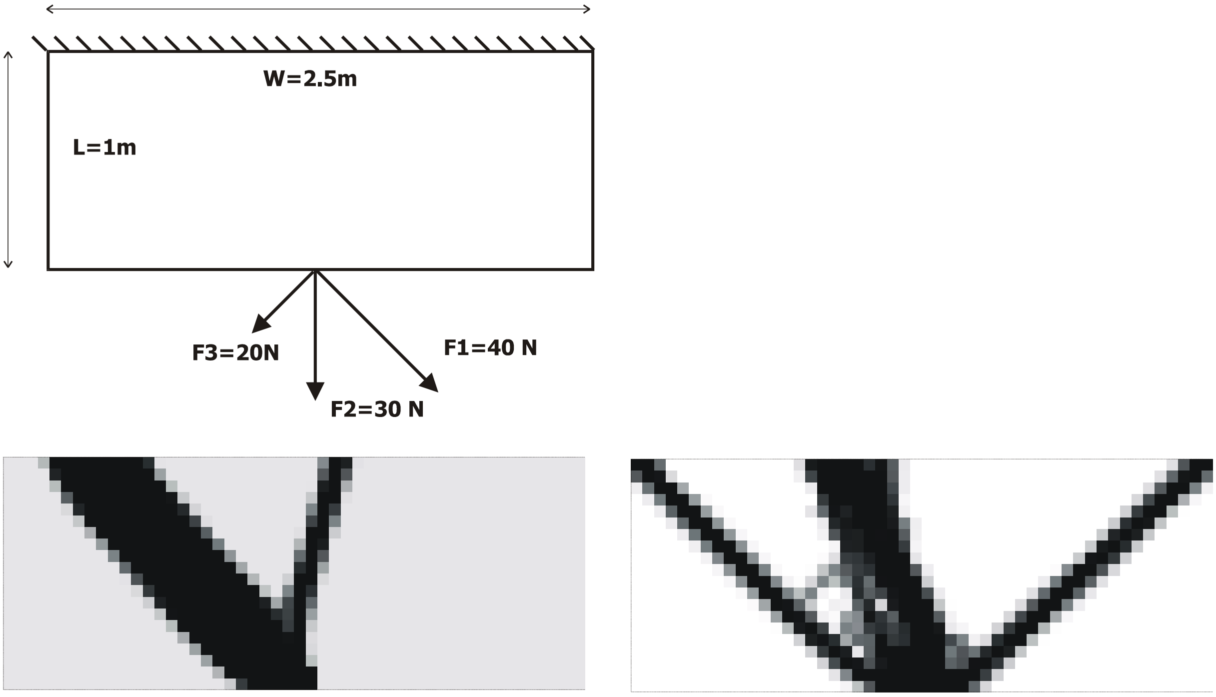


Figure 4: Illustration of the conjecture that topology optimization of stiffness (lower left) and strength (lower right) differ when several load cases are considered.

Despite its academic character, the second application is interesting to illustrate clearly the conjecture that for several load cases, different material and different stress limits in tension and compression, stiffness and strength designs can differ significantly. The design problem is sketched in Fig. 4a (upper). The sizes and material data of the benchmark are normalized: L=1 m, W=2.5 m, E=100N/m², υ=0.3. The design domain is meshed with 50 x 20 finite elements. This means that for local stress constraints one has to deal with 3 x 1000 restrictions. Three load cases are applied independently in the mid point of rectangular domain supported along its upper part. In a first step one solves the problem of the minimum of the maximum of three compliances with volume constraint equal to 25% of the design domain area. The optimal topology is a 2-bar truss (see Fig 4b lower left). Compliances for the 3 load cases are 73.3 Nm, but stress level is quite high. Using von Mises criterion, maximum value of the local criterion varies from 228 N/m² to 571 N/m² per load case. In a second optimization run one consider local von Mises stress constraints with a stress limit of 150 N/m² and one gets the topology of Figure 4c (lower right) which is a 3-bar truss. The compliances of the minimum stress design solution are a bit bigger than minimum compliance solution (91.2 Nm, 45.6 Nm and 45.0 Nm respectively for load cases 1, 2 and 3). Therefore on-going research is devoted to develop new efficient method to take into account the actual problem constraints since the preliminary design in order to capture the really optimal topology of the structure.

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2.4 Optimization of welding spots

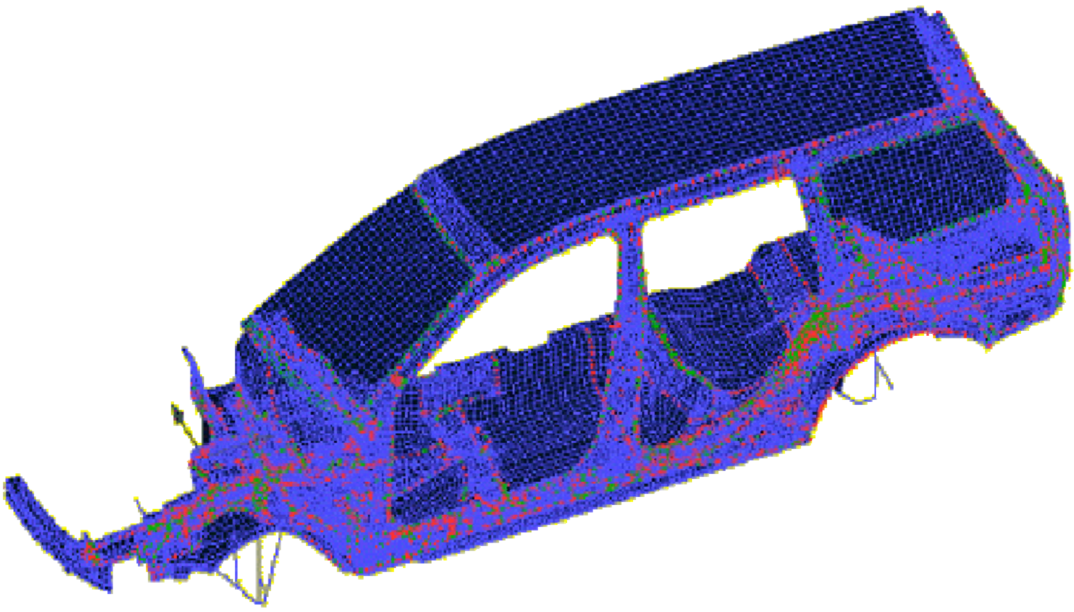

Figure 5: Optimization of spot welding for assembly of an automotive car body

Topology optimization can also be applied to various problems as assembly problems. The formulation of the variable density allows modifying the stiffness properties of welding spots in order to make them appear or disappear from the assemblage. Finding the smallest welding spot subset from a huge set of possible spots is a quite challenging problem without the help of automatic design tools as mathematical programming. Its application to assemblage of car body parts using welding robots is straight forward. This problem was even solved in purely discrete 0/1 formalism by Muriel Beckers [BEC99]. Her work was the basis for an industrial application in Boss Quattro coupled with NASTRAN by Flores [FLO98]. The problem writes as follows: minimise the maximum compliance aver several load cases while satisfying displacement constraints under each load cases and a resource constraint over the number of welding spots. Because it makes sense to have continuously variable density of the welding spots, one can calculate sensitivities for the pseudo density of the welding spots with SOL200 procedure in NASTRAN. Then a discrete valued optimization algorithm based on Fleury's dual optimizer [FLE93b] is used to solve the problem. Fig.5 from [FLO98] illustrates the capabilities of the approach on a half car body structure. Red points are the welding spots retained by the optimization algorithm. Current issue is devoted to integrate manufacturing constraints into the design problem because the time to visit all the spots can change drastically the solution. Thus including the simulated robot manufacturing time as a constraint will improve further quality of predictions.

3 Conclusions and perspectives

After 15 years of active research topology optimization has now become an industrial tool for preliminary design. Several commercial tools are available on the market. Its application to car body optimization has major advantages. Optimal material distribution can suggest innovative and high performance concepts avoiding long time iterative and intuitive processes. Design can be carried out on the basis of rationale and mathematical arguments while engineers can focus on defining the design constraints of his/her problem.

Major on-going and future challenges of topology optimization are devoted to improving the application scope of topology optimization: New applications are concerned with multidisciplinary and recently also to multiphysic problems. For structural problems one should also take care of local constraints that are generally ignored because of the computer time required to solve applications. Manufacturing considerations should also be included as soon as possible in the design loop since they can also modify completely the nature of the optimal solution. However these constraints are extremely difficult to cater with the material distribution representation of the structure.

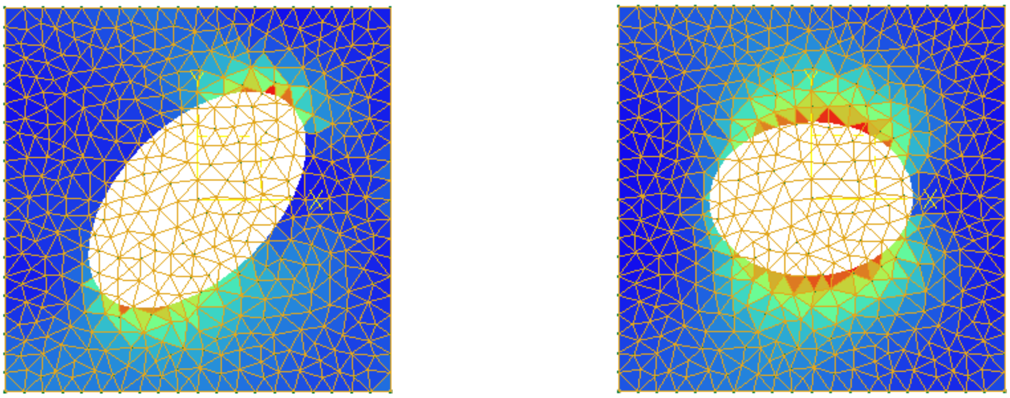

Figure 6: Generalized shape optimization using Level description and XFEM [DUY06]

In order to find a mid way between shape and topology optimization in order to be able to control easierly local constraints and manufacturing considerations, recent work [DUY06] has been devoted to introduce the geometry while not loosing the flexibility of topology optimization. The Level Set description offers such capabilities, allowing geometric entities disappearing or merging. In order to work with fixed mesh like in topology optimization one has to use non conforming finite element techniques as the novel eXtended Finite Element Method. The two elements are the basics of the new research lead at University of Liège using XFEM and level Set description to complement topology optimization and shape optimization.

4 Formula symbols and indices

$f_l(x)$: lth objective function to be minimized

$g_j(x)$: jth design constraint restricting a response function

$\overline{g}_j$: upper bound upon the jth response function

$h_k(x)$: kth equality constrain

$f_l(x)$: objective function to be minimized

x_i design variable i

$\underline{x}_i$ minimum gauge on ith design variable

$\overline{x}_i$ maximum gauge on ith design variable

o number of objective functions

m_i number of inequality design constraints

m_e number of equality constraints

n number of design variables

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